Modelling and Pricing
Hybrid bonds

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Overview

- Common features of hybrid securities
- Pricing hybrid bonds
  - The multiple method
  - SG’s quantitative approach
- Relative value analysis
- Appendices
  - Characteristics of existing issues
  - Sensitivity analysis and stress-testing of the model
  - Technical details on the model
Common features of hybrid bonds
Subordination, and Coupon deferral

- Subordination of capital
  - Hybrid securities are subordinated to Senior debt in case of a default or a restructuring
  - Hybrid securities are typically senior against common equity

- Coupon deferral options: each structure is different
  - Optional deferral
    - In case no dividends or share buy backs
  - Mandatory deferral
    - Usually based on an earnings-related trigger or a solvency ratio (for financial companies)
Maturity, and Extension

- Hybrid bonds are usually long dated, or perpetual…

- … but can be called by the Issuer:
  - Usually callable after 10 years on every payment date
  - A step-up compensates the investor for the extension risk following the first call date

- Pricing the call option requires a dynamic framework
Pricing hybrid bonds
A basic approach: the multiple method

- Each hybrid is compared with its CDS which quantifies its credit risk
- New hybrids can be valued by applying the average multiple in the market
SG’s model: a three-step approach for pricing hybrid bonds

- Simulate all possible future scenarios on credit spreads and default by using:
  - The credit curve of the issuer
  - Assumptions on spread volatility

- Determine the company’s decision for each of these scenarios
  - Coupon deferral
  - Extension

- Assign a probability to each of these scenarios and price the product
Simulate all scenarios on credit spreads

- Use the CDS curve to estimate a base case scenario on spreads
  - CDS curves are extended above the 10y maturity by using the long-term bond market

- Use a spread volatility to simulate all possible deviations around this base case scenario on spreads
  - Spread volatility is taken from the index option market
  - It is adjusted for maturity and correlation mismatch
Simulate all scenarios on default

- Default probabilities are implied from the CDS curve

![CDS Maturity vs. CDS Spd (bp)](image1)

- Recovery rate assumptions are needed to compute default probabilities and default outcomes
  - Senior CDS recovery = 40%
  - Hybrid bond recovery = 0%
Company decision for each scenario

- **Coupon risk**
  - Mandatory vs. Optional deferral
  - Cumulative vs. Non cumulative deferral

- **Extension risk**
  - At each possible redemption date, the company decides whether it wants to call the security depending on the then current price of the security
  - Our model includes a “reputation cost” which prevents the issuer from calling the bond as soon as it rationally makes sense
**Coupon risk: Mandatory deferral**

- Simulate a financial ratio
  - At any interest payment date, coupon is automatically deferred if a given financial ratio stands below threshold
  - We simulate the ratio by a mean-reverting process fitted to historical data
  - Financial ratio is correlated with credit spreads
- Compute the probability to hit the deferral trigger
- Adjust the probability by a risk premium
  - Mandatory deferral risk is similar to default risk
  - It is adjusted by the same risk premium as the default risk premium in the credit market
**Coupon risk: Optional deferral**

- Modelling the issuer’s behaviour
  - Optional deferral is linked to the company’s health

- Coupons are deferred when short-term spreads reach a given threshold
  - 700bp for corporate hybrids

- Historical data on corporates which omitted dividend payments shows that these are realistic thresholds
  - Although sometimes, dividend has not been restored in spite of a sharp tightening of spreads...
  - ... while in other cases, dividend payments were maintained despite a credit crisis
**Coupon risk: Cumulative vs. Non cumulative deferral**

- Non cumulative: coupons are cancelled

- Cumulative: two possible scenarios:
  - Coupons are deferred and then paid back later
    - Losses only come from interests on deferred interests
    - **Assumption:** discarded in our model
  - Coupons are deferred and then the company defaults
    - Deferred interests are lost
    - **Assumption:** two years of deferred interests before default
Extension risk

- At each possible redemption date, the company decides whether it wants to call the security depending on its price
- Reputation cost
  - The company has to face a reputation cost upon extending the security
  - This cost is an additional step-up paid on each coupon date
Pricing model

- A Partial Differential Equation (PDE) enables assigning a probability to each scenario and to compute the Net Present Value of the product in each case.

- Numerical method...
  - The hazard rate model: \( d\lambda_t = \lambda_t (\theta(t) dt + \sigma dW_t) \)
  - Using a binomial tree calibration to extract \( \theta(t) \) from the market
  - Using a bootstrapped IR curve to extract \( r(t) \) from the market
- … based on a PDE grid
  - Diffusing only \( \ln(\lambda) \) (IR and ratio risks are considered independent)
  - Finite differences to approximate first and second order spatial derivatives
  - Von Neumann conditions
  - The PDE:

\[
0 = \frac{\partial V}{\partial t} + \frac{\partial V}{\partial X} (\theta(t) - \frac{\sigma^2}{2}) + \frac{\sigma^2}{2} \frac{\partial^2 V}{\partial X^2} - V(r + e^X) + V_{\text{default}} e^X
\]
Relative value analysis of corporate hybrids
Which indicator for corporate hybrid relative value?

- Subordination risk is the major source of risk between hybrids and senior CDS
  - This is the reason why we use senior recovery rate as a relative value indicator
  - Hybrid recovery rate is set to 0% while senior recovery rate is adjusted to fit to market spreads
The pricing model takes into account senior recovery rates for valuing a hybrid bond:

- Senior recovery rate can be adjusted to market spreads.
- Final model spread is a theoretical spread based on a basket average senior recovery rate.
Pricing corporate hybrids: the multiple approach

- Average multiple was 4.06x in the corporate hybrid market on 12 February 2007

Relative value analysis of corporate hybrids based on multiple vs. CDS

<table>
<thead>
<tr>
<th></th>
<th>Interpolated CDS</th>
<th>Market ASW spd</th>
<th>Multiple</th>
<th>Fair ASW spd</th>
<th>Spd pick-up</th>
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Average multiple used for the fair spd
Pricing corporate hybrids: SG’s model results

- Implied recovery rate was 33% on May 29, 2007 on the corporate hybrid market assuming a 200bp reputation cost

<table>
<thead>
<tr>
<th>Company</th>
<th>Market ASW spread</th>
<th>Interpolated CDS</th>
<th>Implied senior recovery rate</th>
<th>Fair ASW spread</th>
<th>Spread pick-up against senior debt</th>
<th>Last week spread pick-up</th>
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</table>

Source: SG Credit research

Average implied senior recovery rate: 33%

Top picks against CDS

Roughly fairly priced

Most expensive issues against CDS

Spread pick-up

Last week spread pick-up
**Risk impacts for each issue**

- Each hybrid is priced at its fair value (using the average senior recovery rate) and risk factors are then removed one by one to compute their impact on the spread.

- Extension risk has an average fair value of 59bp, coupon risk of 40bp, and subordination risk of 22bp.

**Split of the fair spread of corporate hybrids**

<table>
<thead>
<tr>
<th>Company</th>
<th>Fair ASW spread</th>
<th>Coupon deferral impact</th>
<th>Extension impact</th>
<th>Subordination impact</th>
<th>Bullet bond spread</th>
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Source: SG Credit research
Relative value analysis of subordinated financials
Adapting the model to subordinated financials

- Assumptions for subordinated financials
  - Subordinated CDS recovery rate: 20%
  - Recovery rate for LT2: 20%, for UT2: 10%, for T1: 0%

- The model is the same except for dated insurance bonds
  - They have the same subordination as CDS (LT2) therefore they are not impacted by the CDS recovery rate in the model
  - The reputation cost is used as the adjustment parameter in the model instead of the CDS recovery rate
Model results for subordinated financials, May 29, 2007

Bank Tier 1 issues with step-up

Dated subordinated insurance

Bank Tier 1 issues without step-up

Undated subordinated insurance
Conclusion
An exhaustive framework for valuing hybrid securities

- The model developed by SG is a complete and mathematically coherent framework for valuing hybrid securities
  - Taking into account all sources of risk affecting hybrid products
  - A quantitative and fundamental approach at the same time

- It can be used to:
  - Find the fair value of options embedded in each structure
  - Analyse the relative value between issues

- Results are updated every week on existing corporate, insurance and bank subordinated issues